RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

4728/01

MATHEMATICS

Mechanics 1

FRIDAY 6 JUNE 2008

Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

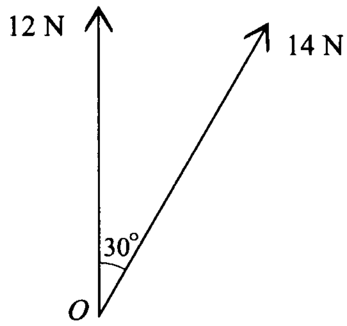
[Turn over

1 A car of mass 900 kg is travelling in a straight line on a horizontal road. The driving force acting on the car is 600 N, and a resisting force of 240 N opposes the motion.

(i) Show that the acceleration of the car is $0.4\,\mathrm{m\,s^{-2}}$. [2]

(ii) Calculate the time and the distance required for the speed of the car to increase from $5\,\mathrm{m\,s^{-1}}$ to $9\,\mathrm{m\,s^{-1}}$. [4]

2

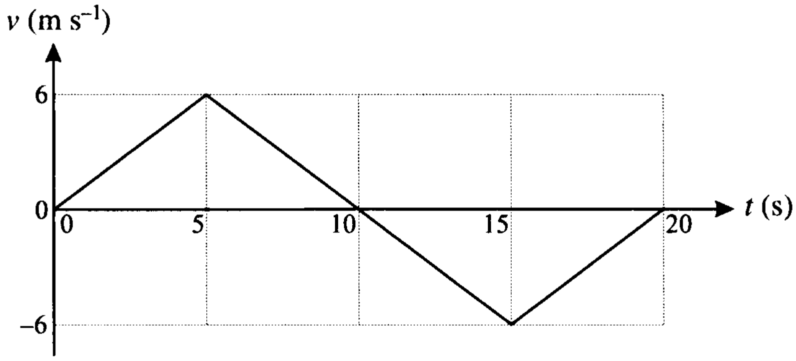


Two horizontal forces act at the point O. One force has magnitude 12 N and acts along a bearing of 000°. The other force has magnitude 14 N and acts along a bearing of 030° (see diagram).

(i) Show that the resultant of the two forces has magnitude 25.1 N, correct to 3 significant figures. [5]

(ii) Find the bearing of the line of action of the resultant. [3]

3



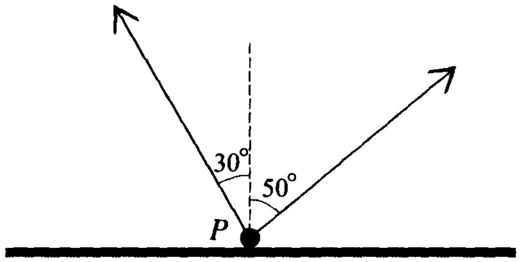
An athlete runs in a straight line from point A to point B, and back to point A. The diagram shows the (t, v) graph for the motion of the athlete. The graph consists of three straight line segments.

(i) Calculate the initial acceleration of the athlete. [2]

(ii) Calculate the total distance the athlete runs. [3]

(iii) Calculate the velocity of the athlete when $t = 17$. [3]

4



A particle P of weight 30 N rests on a horizontal plane. P is attached to two light strings making angles of 30° and 50° with the upward vertical, as shown in the diagram. The tension in each string is 15 N, and the particle is in limiting equilibrium. Find

(i) the magnitude and direction of the frictional force on P, [3]

(ii) the coefficient of friction between P and the plane. [5]

5 A railway wagon A of mass 2400 kg and moving with speed 5 m s^{-1} collides with railway wagon B which has mass 3600 kg and is moving towards A with speed 3 m s^{-1}. Immediately after the collision the speeds of A and B are equal.

(i) Given that the two wagons are moving in the same direction after the collision, find their common speed. State which wagon has changed its direction of motion. [5]

(ii) Given instead that A and B are moving with equal speeds in opposite directions after the collision, calculate

(a) the speed of the wagons,

(b) the change in the momentum of A as a result of the collision.

[5]

6 A model train travels along a straight track. At time t seconds after setting out from station A, the train has velocity v m s^{-1} and displacement x metres from A. It is given that for $0 \leqslant t \leqslant 7$

$$x = 0.01t^4 - 0.16t^3 + 0.72t^2.$$

After leaving A the train comes to instantaneous rest at station B.

(i) Express v in terms of t. Verify that when $t = 2$ the velocity of the train is 1.28 m s^{-1}. [3]

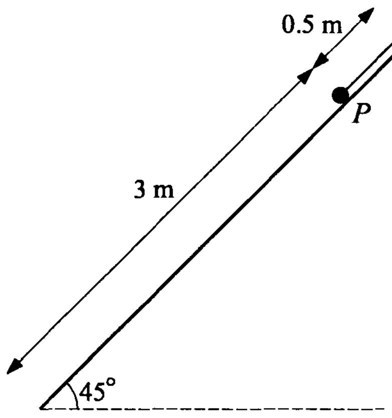
(ii) Express the acceleration of the train in terms of t, and hence show that when the acceleration of the train is zero $t^2 - 8t + 12 = 0$. [3]

(iii) Calculate the minimum value of v. [4]

(iv) Sketch the (t, v) graph for the train, and state the direction of motion of the train when it leaves B. [4]

(v) Calculate the distance AB. [2]

7



Two particles P and Q are joined by a taut light inextensible string which is parallel to a line of greatest slope on an inclined plane on which the particles are initially held at rest. The string is 0.5 m long, and the plane is inclined at 45° to the horizontal. P is below the level of Q and 3 m from the foot of the plane (see diagram). Each particle has mass 0.2 kg. Contact between P and the plane is smooth. The coefficient of friction between Q and the plane is 1. The particles are released from rest and begin to move down the plane.

(i) Show that the magnitude of the frictional force acting on Q is 1.386 N, correct to 4 significant figures. [2]

(ii) Show that the particles accelerate at 3.465 m s^{-2}, correct to 4 significant figures, and calculate the tension in the string. [5]

(iii) Calculate the speed of the particles at the instant when Q reaches the initial position of P. [2]

At the instant when Q reaches the initial position of P, Q becomes detached from the string and the two particles travel independently to the foot of the plane.

(iv) Show that Q descends at constant speed, and calculate the time interval between the arrival of P and the arrival of Q at the foot of the plane. [7]

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1(i)	900a = 600 -240 a = 0.4 ms^{-2} AG	M1 A1 [2]	N2L with difference of 2 forces, accept 360
(ii)	9 = 5 + 0.4t t = 10 s $9^2 = 5^2$ + 2x0.4s s = 70 m	M1 A1 M1 A1 [4]	v = u + 0.4t or v = u +(cv 0.4)t or s=(u+v)t/2 or s=ut+0.5xcv(0.4)t^2

2(i)	Resolves a force in 2 perp. directions Uses Pythagoras R^2 = (14sin30)^2 + (12+14cos30)^2 {or R^2 = (12sin30)^2 + (14+12cos30)^2 } R = 25.1 AG	M1* D*M1 A1 A1 A1 [5]	*Uses vector addition or subtraction* *Uses cosine rule R^2 =* *14^2 + 12^2 -* *2x14x12cos150* cso (Treat *R^2 =14^2+12^2 + 2x14x12cos30* as correct)
(ii)	Trig to find angle in a valid triangle tanB=7/24.1,sinB=7/25.1,cosB=24.1/25.1 B = 016, (0)16.1^o or (0)16.2^o	M1 A1 A1 [3]	Angle should be relevant *sinB/14 = sin150/25.1*. Others possible. Cosine rule may give (0)16.4, award A1

3(i)	a = 6/5 a = 1.2 ms^{-2}	M1 A1 [2]	Acceleration is gradient idea, for portion of graph Accept 6/5
(ii)	s = (6x10/2) x2 {or (6x5/2) x4} s = 60 m	M1 M1 A1 [3]	Area under graph idea or a formula used correctly Double {Quadruple} journey
(iii)	 v = -6 + 1.2(17-15) v = -3.6 ms^{-1}	M1 A1 A1 [3]	v=u+at idea, t not equal to 17 (except v=1.2t-24) 0 = v + cv(1.2)(20-17), v^2 -2.4v -21.6 = 0, etc ***SR*** v=3.6 neither A1, but give both A1 if final answer given is -3.6

4(i)	 F = 15sin50 - 15sin30 = 3.99 N Left	M1 A1 B1 [3]	Difference of 2 horizontal components, both < 15 Not 4 or 4.0 Accept reference to 30 degree string May be given in ii if not attempted in i
(ii)	 R = f(30, 15cos50, 15cos30) R = 30-15cos50-15cos30 μ = 3.99/7.36(78) μ = 0.541 or 0.542 or 0.543	M1 A1 A1 M1 A1 [5]	Equating 4 vertical forces/components 30g is acceptable =7.36(78..), treat 30g as a misread Using F = μR, with cv(3.99) and cv(7.36(78..)) Accept 0.54 from correct work, e.g. 4/7.4

5(i)	2400x5 - 3600x3 2400v + 3600v 2400x5 - 3600x3 = 2400v + 3600v v = 0.2 ms^{-1} B	B1 B1 M1 A1 B1 [5]	Award if g included Award if g included Equating momentums (award if g included) Not given if g included or if negative.
(ii)(a)	+/-(-2400v + 3600v) 2400x5 - 3600x3 = -2400v + 3600v v = 1 ms^{-1}	B1 M1 A1	*No marks in(ii) if g included* Equating momentums if "after" signs differ Do not accept if - sign "lost"
(b)	I = 2400 x (5+/-1) or 3600 x (3+/-1) I = 14400 kgms^{-1}	M1 A1 [5]	Product of either mass and velocity change Accept -14400

6(i)	$x = 0.01t^4 - 0.16t^3 + 0.72t^2$.		
	v = dx/dt	M1	Uses differentiation, ignore +c
	$v = 0.04t^3 - 0.48t^2 + 1.44t$.	A1	or $v = 4(0.01t^3) - 3(0.16t^2) + 2(0.72t)$
	v(2)=1.28 ms^{-1} AG	A1	Evidence of evaluation needed
		[3]	
(ii)	a = dv/dt	M1	Uses differentiation
	$a = 0.12t^2 - 0.96t + 1.44$	A1	or $a = 3(0.04t^2) - 2(0.48t) + 1.44$
	$t^2 - 8t + 12 = 0$ AG	A1	Simplifies $0.12t^2 - 0.96t + 1.44 = 0$, (or verifies the roots of QE make acceleration zero)
		[3]	
(iii)	(t - 2)(t - 6) = 0	M1	Solves quadratic (may be done in ii <u>if used to find v(6)</u>)
	t = 2	A1	Or *Factorises v into 3 linear factors* M1
	t = 6	A1	$v = 0.04t(t-6)^2$ A1 *Identifies t=6* A1
	v(6) = 0 ms^{-1}	B1	Evidence of evaluation needed
		[4]	
(iv)		B1	Starts at origin
		B1	Rises to single max, continues through single min
		B1	Minimum on t axis, non-linear graph
	Away from A	B1	
		[4]	
(v)	AB $=0.01\text{x}6^4 - 0.16\text{x}6^3 + 0.72\text{x}6^2$	M1	Or integration of v(t), with limits 0, 6 or substitution,
	AB = 4.32 m	A1	using cv(6) from iii
		[2]	

7(i)	(R=)0.2x9.8cos45	M1	Not F = 0.2x9.8cos45 or 0.2x9.8sin 45 unless followed
	F=1xR=1x.2x9.8cos45=1.386 N AG	A1	by (eg) Fr =1x F = 1.386 when M1A1
		[2]	
(ii)	Any 1 application of N2L // to plane with correct mass and number of forces	M1	Must use component of weight
	0.4a=0.2gsin45+0.2gsin45-1.38(592..)	A1	
	a = 3.465 ms^{-2} AG	A1	
	0.2a = 0.2gsin45 - T or		Accept with 3.465 (or close) instead of a
	0.2a = T +[0.2gsin45 - 1.38(592..)]	M1	Accept omission of [term] for M1
	T = 0.693 N	A1	Accept 0.69
		[5]	
	OR		
	Any 1 application of N2L // to plane with correct mass and number of forces		Must use component of weight
	0.2a = 0.2gsin45 - T or	M1	Either correct
	0.2a = T +[0.2gsin45 - 1.38(592..)]	A1	Both correct. Accept omission of [term] for A1 only
	Eliminates a or T	M1	
	a = 3.465 ms^{-2} AG	A1	
	T = 0.693 N	A1	
(iii)	$v^2 = 2 \text{ x } 3.465 \text{ x } 0.5$	M1	Using $v^2 = 0^2 + 2\text{xcv}(3.465)\text{s}$
	v = 1.86 ms^{-1}	A1	
		[2]	
(iv)	For Q		
	(0.2)a = (0.2)gsin45 - (1)(0.2)gcos45.	M1	Attempting equation to find a for Q
	a=0 [AG]	A1	Accept from 0.2gsin45 - 1.386
	T = (3/1.86) =1.6(12)	B1	Accept 2 sf
	For P		
	a = 9.8sin45	B1	a= 6.93
	$2.5 = 1.86(14..)t + 0.5 \text{ x } (9.8\sin45)t^2$	M1	Using $2.5 = \text{cv}(1.86)t + 0.5\text{cv}(6.93)t^2$ [not 9.8 or 3.465]
	t = 0.6(223)	A1	Accept 1sf
	time difference 1.612 - 0.622 = 0.99(0) s	A1	Accept art 0.99 from correct work
		[7]	